

A SHORT SUMMARY OF THE STRUCTURAL DYNAMICS RESEARCH
CARRIED OUT AT STANFORD UNIVERSITY UNDER THE SPONSORSHIP OF THE
OFFICE OF NAVAL RESEARCH

by

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Introduction:

During the seven year period of the existence of the contract, N6-Ori-154, T.O. #1, a total of 18 reports, falling into four groups, have been written to cover the work accomplished. Seven of these deal with the transverse, elastic vibrations of various beams supporting several types of moving loads; five relate to the non-linear structural response of simplified models caused by suddenly applied forces; two have been concerned with the hydro-dynamic loads imposed on rigid, cylindrical containers suffering sudden displacements; two deal with the phase-plane method of solving problems, and two reports have not been specifically related to each other or to the already mentioned groups. It is the object of this summary to give a short perspective of the work performed under the O.N.R. sponsorship.

The enlightened attitude of the sponsor toward research, namely that it can be specified only in a general direction, has been much appreciated by all the project workers. The feeling has prevailed that our studies were neither circumscribed by external demands, nor were they goaded on by rigid time limitations. In other words, academic freedom to study the general field of structural dynamics has been enjoyed. Under these circumstances it has been possible to make use of numerous graduate students and to give them a chance to participate in the work according to their abilities. The research has resulted in the publication of 11 technical papers of general interest, distributed in three technical journals.

Throughout the seven year period we have been allowed to define the word "structure" in a general way. It has not been limited to mean

combinations of the structural elements commonly dealt with in the field of structural engineering. In fact, no work has been done directly on structural elements such as I-beams, reinforced concrete, etc. Instead we have experimented on models designed to simulate the behavior of various systems that sometimes had entirely different physical appearances from the models.

Our approach to a structural problem has been to apply pencil and paper analysis as far as reasonably practicable; then, if the ultimate answer involved too much calculation, we have supplemented or possibly even replaced calculations by planned laboratory experimentation employing mechanical models suitably loaded. This technique has been used at Stanford since 1931 when our first "indestructible" model of a large building was created.

The analogy of indestructible models with their destructible prototypes is limited to a simple correspondence between their respective mass and rigidity characteristics plus equally simple correspondences with the applied disturbance intensities and shapes. Frictional effects sometimes have been taken into consideration in an approximate way; at other times plastic hysteresis has been idealized to the point where the restorative elements of the model are able to represent this effect fairly adequately. Essentially an "indestructible" model has no stress similitude to satisfy; it is solely a deformation model. Its correspondence with reality is based on the assumption that the stress-strain properties of the prototype can be calculated, estimated, or even tested by auxiliary means, which are not too difficult to devise if the strain rates are moderate.

When research was begun in 1946, studies of non-linear systems were discussed with the Head of the Mechanics Branch of O.N.R., but it was

decided to place more emphasis on ideally elastic systems at first. Accordingly the seven years of research may be divided into two overlapping periods of approximately equal duration, the first of which deals mainly with linear problems, and the second with non-linear. During the last four years it has become increasingly apparent that engineering interest in dynamically loaded structures not only extends well into the plastic regions preceeding their collapse, but actually centers around the question of whether or not collapse will take place.

It may be argued that tests on "indestructible" models will be inferior to tests on destructible ones where the intrinsic properties of the model materials are real and consequently subject to failure. It must be admitted that this reasoning is valid if all similitude conditions can be satisfied and if one is contented with the yield of one single, specific test of each model specimen. Thus, if the dynamic loading intensity is large enough to produce plastic deformations, the usefulness of a stress-strain similitude model may be limited to a single test in which its material initially is presumably in the fabricated, virgin condition. In other words, the history of the stress-strain similitude model is of importance. For reasons of cost alone it is therefore desirable to investigate what can and what cannot be answered by an "indestructible", deformation model of a structure.

In general a deformation model can be made to represent the integrated properties of an entire structure up to its point of collapse if the structure collapses as a whole and in a predictable manner. This means that secondary collapses of a roof, as well as rigidity changes in the prototype's structural

members for instance due to torsional effects, cannot be taken into account adequately in a deformation model.

In spite of this limitation, it is believed that deformation models can teach us a great deal about the behavior of dynamically loaded structures. In Figure 1 of T. R. #18, included here, a model of a four-story structure is represented diagrammatically. The twelve bars are pin connected and derive their stability in the plane of the drawing from the two systems of helical springs fastened to the four brake drums in which slippage will take place when the elastic spring forces reach an adjustable value. The stiffnesses of the large springs are seen to correspond to the elastic properties of the structure, while the brake drum force together with the stiffnesses of the small springs create the "elastic limit" and the ensuing plastic "slope" of the structure's restoring force element. Instability of the model due to vertical forces is inherent in the construction.

The following conclusions in regard to "indestructible" models seem to be justified:

- a) Economical experimental studies designed to answer the question: "Will a given structure collapse as a whole when exposed to a given dynamic load", may be carried out by "indestructible" deformation models.
- b) Idealizations necessary in the design and operation of deformation models are fully as realistic as those necessary in purely computational studies.
- c) Changes in deformation models and in their types of loading can readily be made since the models may be stacked or otherwise

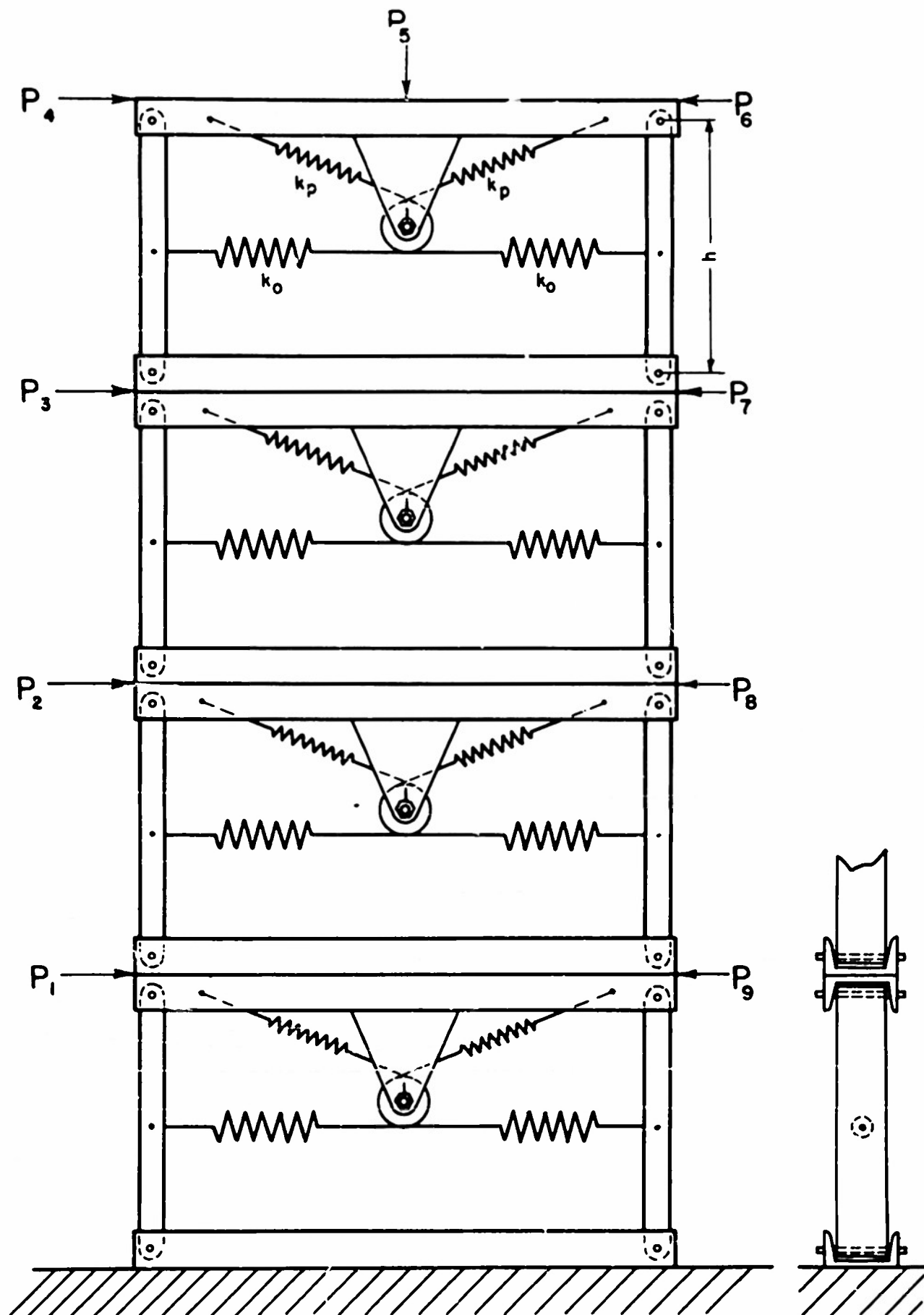


FIG. 1

coupled, and since the force application devices may be made self contained units governed by a master timing device.

A more or less chronological account of the work described in the 18 reports will now be given in bold strokes.

GROUP I

I The Moving Loads on Beams

The seven technical reports in the above series are:

- T.R. #1 Transverse Vibration of a Two-Span Beam Under the Action of a Moving Force (March 1948)
- T.R. #4 Transverse Vibration of a Two-Span Beam Under the Action of a Moving Alternating Force (March 1949)
- T.R. #5 Natural Frequencies of Continuous Beams of Uniform Span Length. (March 1949)
- T.R. #7 Transverse Vibration of One and Two-Span Beams Under the Action of a Moving Mass Load. (October 1949)
- T.R. #10 Transverse Vibration of One and Two-Span Beams Under the Combined Action of A Moving Mass Load and A Moving Alternating Force. (July 1950)
- T.R. #12 Transverse Vibration of One and Two-Span Beams Under the Action of a Moving Combined Load Consisting of A Spring-Borne Mass and an Alternating Force. (March 1951)
- T.R. #15 Transverse Vibration of Multi-Span Continuous Beams Under the Action of a Moving Alternating Force. (September 1951)

Analytical solutions for the dynamics of the one span, uniform beam were started as early as 1847 and fall into three categories according to the simplifying assumptions made:

- I The beam has negligible mass in comparison with the mass of the concentrated moving load.
- II The mass of the concentrated moving load is negligible in comparison with the mass of the beam.
- III Both masses are considered, and the moving load is concentrated.

Assumption I, originally made by Professor R. Willis in 1847, led to a solution by Sir G. G. Stokes in 1849. Extensions of the problem based on this assumption have not been made by us as it is realistic only in the case of short single span beams.

Of the three assumptions No. II gives the simplest solution since the natural frequencies of such a system are constant or independent of the load's location. T.R. #1 and #4, deal with this case for a two-span beam of equal spans. While the two span system is not different in theory from the previously solved single span, it is definitely more involved as to determinations of constants. There are twice as many modes and half of them are of non-sinusoidal shape. These two reports served to develop an adequate experimental procedure and to indicate what agreement might be obtained between experiment and theory. Moreover T.R. #4 clarifies the concepts of: lower, stationary, and upper, translatory resonances.

Theoretical solutions for a single span beam based on assumption III are complicated by the fact that the natural frequencies and mode shapes of the beam-load system depend on the variable location of the moving load. A solution employing successive approximations was published by Jeffcott in 1929. This was followed by a more general study by Steuding in 1935 and by Schallenkamp in 1937. Lately A. Hillerborg in Sweden has made an experimental study of the one span beam.

Our studies based on assumption III start out with a preliminary consideration of the natural frequencies of uniformly loaded continuous beams of many equal span lengths, T.R. #5. This is followed by T.R. #7, #10, and #12, all relating to one and two span beams with increasingly composite

types of moving loads. Thus T.R. #7 shows satisfactory agreement between Schallenkamp's theoretical solution and our experiments on the single span beam; it extends the experimental studies to the two span system and gives performance curves for the case of a concentrated mass moving along the beam with constant velocity.

In T.R. #10 a study is made of the situation when a concentrated mass load coincident with an alternating force load moves along the beam with constant velocity. To make the system approach reality, viscous damping is introduced at the beam supports. In this case resonance effects with the alternating force become pronounced.

Finally in T.R. #12 the previous beam systems are acted upon by a concentrated, spring-borne, mass load coincident with the moving alternating force load. The beams again have viscous damping concentrated at their supports, and the spring-borne mass has viscous damping proportional to its relative velocity with respect to that of the beam. A "pairing" of resonance peaks, corresponding to the "in phase" and "out of phase" modes of the spring-borne mass with the beam, now becomes pronounced and further complicates the resonance curves, especially for the two span beam where the natural frequencies of the fundamental and second modes are close together. It is therefore not strange that the total phenomenon, involving "pairing" as well as "lower" and "upper" translatory frequencies, becomes so complicated that a final discussion of the performance curves has been based on an assumed "normal distribution" of the velocity of the moving loads. Consequently "normally weighted averages" of dynamic stresses at mid-spans or at the central support have been computed. A consideration of damping

leads to the view that the action of the spring-borne mass is similar to that of a damped dynamic vibration absorber. Its effect is more pronounced for the "out of phase" motion than for the "in phase".

An extension of the previously investigated one and two-span beams to multispans continuous beams is covered by T.R. #15. The study is limited to the action of a moving constant force plus an alternating force and gives experimental results for a four-span continuous beam. Residual, or free vibration spectra of stress at the various mid-spans and supports show gratifying symmetry in regard to peak values and locations. Envelopes of spectra for one, two, and four-span beams are compared, and the influence of friction is found for two specific values of damping. Finally a theoretical extension of the residual vibration's fundamental mode stress component for continuous beams up to ten equal spans is given.

This concludes the study of moving loads on beams described in the seven technical reports, but in addition to these, two Laboratory Memoranda for internal distribution have been produced, they are:

L.M.1. The Traveling Load on the Beam on An Elastic Foundation
(May 1952)

L.M.2. A Discussion of the Transverse Vibration of a Single-Span Beam
Under the Action of a Moving, Distributed Mass Load. (June 1952)

Both memoranda show promise of work that might have been expanded into the scope of reports.

GROUP II

The Bi-Linear Restoring Force System

The five technical reports in the above series are:

T.R. #2 Response of an Elastically Non-Linear System for Transient Disturbances. (March 1948)

T.R. #11 and #14 Response of a Yielding Vibratory System to Transient Forcing Functions, (I)(March 1948) and (II)(August 1951)

T.R. #17 Development of a "Deformation Model" of a Building for the Study of Blast Load Effects. (June 1952)

T.R. #18 Further Development and Tests of a Collapsible, Elasto-Plastic Deformation Model. (August 1953)

All of the studies relating to the bi-linear restoring force system employ phase-plane analysis for their theoretical considerations.

In T.R. #2 the simple oscillator experimented upon is as nearly frictionless as possible; its bi-linear restoring elements are of the stiffening type and have almost no hysteresis; its motion is strictly rectilinear. Only three simple ground motions were considered, namely, the "square wave", the "versed sine" and the "skewed-versed sine" type. The last two excitations lend themselves to experimentation, but the "square wave" type was treated by simple theory only.

The oscillator system dealt with in T.R. #11 and #14 had a bi-linear restoration of the softening type; it was assumed to be frictionless, but hysteresis in the restoring element was introduced. Its motion was still rectilinear, and its excitation was limited to the "square wave" type, occasioned either by a ground motion or by a direct force application to the mass. No experimentation was done in connection with this, it being mainly a drafting board study employing the phase-plane technique. Permanent sets of the system resulted from the action of some of the transient loads.

In T.R. #17 a development of a rotatory slippage type of friction brake was begun and carried to a reasonable degree of reliability for the system with strictly rectilinear motion. The same type of friction brake

was then applied to a collapsible oscillator in which the mass at first displaces much more in the horizontal direction than in the vertical; then if the state of collapse is approached the two displacements reach the same order of magnitude, and finally, if collapse occurs, the motion becomes mainly vertical.

For loading purposes the collapsible, bi-linear oscillator may be placed on a mechanically actuated shaking table and blast loadings may be simulated by applying concentrated forces at selected points on the model. The devices used for creating these forces are of the pneumatic type, involving cylinders and pistons. The timing, intensity, shape, and duration of the piston forces are controllable within certain limits and may be made to represent, in a lumped form, the pressure loading of a shock wave initially obtained from either theoretical or experimental studies.

The timing of the operation of a loading device is effected by an electrically operated ball valve, set off by a mechanical sweep that controls all the loading devices. The intensity, shape and duration of the piston force can be regulated by adjusting five independent quantities, namely: pressure and volume of air in the reservoir, initial volume of air in the cylinder, throttling of the flow from reservoir to the cylinder, and bleeding of the air from the cylinder. Two types of bleeding are used - adjustable orifice, and inertia pop-off valve.

Since SR-4 gauges are placed on a ring dynamometer in series with the piston rod, a time record of the force exerted by the device against the model is obtained. The adjustments of the five quantities necessary to simulate the blast force can be made by successive trials; moreover the

final adjustments will take into account the piston and model motion. Thus it is clear that only an "indestructible" type of model would be practical in conjunction with the type of loading device used. In this connection it should be said that the performance of model and devices is remarkably consistent; it can be duplicated again and again within narrow limits.

Tests on a one story structure exposed to a simulated blast load and involving a front, roof and rear pneumatic loading device were carried out and show satisfactory agreement with theoretical calculations made by phase-plane drawings.

T.R. #18 describes the final development of a four story, collapsible, elasto-plastic deformation model. Considerable difficulties with the friction brakes had to be overcome; the recording devices had to be improved; and the general question of consistency in operation had to be answered. Extensive tests on the model in the 1, 2, 3, and 4 story state have been carried out and show that optimum distributions of yield point values in a multistory structure can be found for a given type and distribution of blast loading. The effect of mass is also determined. In Appendix I an interconnection of floors making the model simulate a building with elastic girders is shown.

A year of intensive developmental experimentation with the four story model resulted in a considerable number of improvements and the conclusions that many practical questions about blast resistance can be answered with a minimum of experimental effort.

GROUP III

Hydrodynamics of Cylindrical Containers

T.R. #6 Impulsive Hydrodynamics of Fluid Inside a Cylindrical Tank and of Fluid Surrounding a Cylindrical Pier. (Jan. 1949)

T.R. #6 Hydrodynamic Experiments with Rigid Cylindrical Tanks
Subjected to Transient Motions. (March 1950)

In connection with any type of shock load it is of interest to evaluate the dynamic mass effect of fluid in a partially filled, rigid container. As a consequence of the impulsive character of a shock load, subsequent free vibration, gravitational waves may be set up.

T.R. #6 deals theoretically with the impulsive time era and gives equivalent rigid masses and static mass moments for the fluid. It also gives velocity and displacement distributions inside or outside the rigid boundaries of the cylinder.

T.R. #8 describes experimental work that confirms the theoretical work of T.R. #6 and extends the impulsive phenomenon into the subsequent time era when only gravitational waves exist. Displacement continuity between the two time eras is difficult to satisfy at the fluid boundaries. Experiments are therefore devised to answer the practical question of the magnitude of gravitational waves.

Four model tanks, of approximately 6, 12, 24, and 48 inches in diameter have been studied so as to give an idea of scale effect. The covered tank with small air space is also studied experimentally. The results indicate that the "clearance" necessary to reduce the hydrodynamic mass effect to open tank value is very small. The ground motions used in the experiments were produced by dropping a pendulum against a bumper spring on a shaking table. They were of two types, the single step and the oscillatory.

GROUP IV

Phase-Plane Methods

T.R. #9 Transient Vibration of Linear Multi-degree of Freedom Systems
by the Phase-Plane Method. (June 1950)

T.R. #13 On a General Method of Solving Second Order Differential Equations by Phase-Plane Displacements. (March 1951)

Phase plane-methods in engineering were introduced by J. Lampen in 1935. They have been especially useful in connection with one degree of freedom systems.

T.R. #9 describes the application of phase plane technique to multi-degree of freedom systems using a few specific examples as illustrations.

In T.R. #13 a general method of tackling the linear as well as the non-linear differential equation has been formulated. It deals with a standard form of solution in which all non-linearities in restoration, all linear and non-linear dissipative terms, and all external driving terms are expressed by an operative displacement called delta. A step by step graphical integration is then used for the solution. The accuracy of the method is tested by examples for which known solutions exist, but in the general case the degree of approximation can be known only empirically.

A laboratory report L.R. #3 with the title "Application of the Phase-Plane Delta Method to Non-Linear and to Linear Multi-Degree of Freedom Systems" (June 1952) indicates that an iterative step by step integration by this method does not involve an excessive amount of work.

MISCELLANEOUS

T.R. #3 Steady Forced Vibrations of a Non-Conservative System with Variable Mass. (November 1948)

The analysis relates to a reciprocating pumping system in which a change in mass occurs between the working stroke and the return stroke. Four distinct classes of motion are shown to exist, and magnification factors as functions of forcing frequency have been obtained for a wide

range of parameters. Good agreement between theory and experiment is found except in the immediate vicinity of resonance.

T.R. #16 A Comparative Study of Pulse and Step-Type Loads on A Simple Vibratory System. (January 1952)

The subject covered by T.R. #16 might well have been used as the starting point for our sponsored research. A simple linear oscillator without damping is excited in turn by 17 different types of pulses, many of which present different degrees of skewing. Moreover 8 different step functions have been considered. Response spectra for the forced vibration era as well as for the residual free vibration era have been drawn. The main purpose of the report is to provide a graphical comparison of the oscillator's responses to the following types of excitation: force application to mass, ground displacement, and ground acceleration.

Appraisal of Work Accomplished

In looking over the 18 reports the question is invariably asked "how significant are the results"? If the individual studies had been requested by agencies vitally concerned with the answers that is, 'if the sponsored research had been closely directed in scope by some agency', an assessment by it should have been solicited.

Since this has not been the case, and since the criterion of being "vitally concerned" is an absolute necessity for judging the merits of nearly all research, we propose to submit an appraisal ourselves.

1) Our work on "Moving Loads on Beams" has clarified the perplexing situation relating to multiple span effects. The presented results deal specifically with equal-span systems and in particular with the two-span. Stress factors have been obtained for the uniform motion of a single

concentrated load of the constant force, constant mass, or spring mass, plus alternating force type. It is regrettable that the case of several concentrated loads moving with the same constant speed, or the uniformly distributed, moving load of finite length could not be entered into in sufficient detail to warrant a report. The question of localized damping at the bridge supports and at the spring-borne mass has been touched upon too briefly to allow us to give sufficient data for design purposes. Moreover the effects of yielding supports, or of an elastic foundation for the beams have been left out. Our study therefore cannot be called exhaustive. The reasons for abandoning the moving load studies before the termination of the contract were a) the non-linear dynamics field had caught our interest, and b) the Sponsor suggested that the moving load studies were not of vital interest to the U.S. Navy.

2) Our studies of the Bi-Linear Restoring Force System were begun before the work described under 1) had been terminated. Extensive use of the phase plane technique in connection with experimentation on simple models characterize all the reports. The development of a general technique described as the Phase-Plane-Delta method resulted from the techniques which in the earlier reports followed the pattern of Lamⁿoen. The principal achievement of our non-linear studies is, in our estimation, the physical development of a simplified, non-linear, hysteretic, and collapsible building model. It is true that the model we have constructed assumes the existence of infinitely rigid girders and therefore couples a story only with the one below and with the one above, but it has been shown in T.R. #18, that a relatively simple modification of the model will extend the coupling

of a story to all the other stories in the building thereby achieving the effect of having girder systems of finite stiffnesses. The question of whether or not the mechanical model developed by us is inferior, equal, or superior to possible electrical analogies or to digital computational methods has not been entered into since from a research point of view several avenues of approach are desirable provided that their limitations as well as their strengths are recognized. Our model's principal limitations are: a) presence of some uncontrollable friction in the links of the frame and in the loader pistons, b) a slight inconstancy of the slippage brakes over a long period of time, c) a limited versatility of the pneumatic loaders. Its strength consists in its immediate physical character and in the ease with which intelligible experimentation can be carried out directly.

3) The work on Hydrodynamics of Cylindrical Containers has been carried to a point where application to design problems can be made from the information given in the reports. The limitation of a rigid container is admittedly such that practical engineering judgment must be exercised in using the contents of the reports for problems relating to an elastic container.

4) Altogether 36 people have worked on the contract since its beginning. Its educational contribution therefore has been considerable in furnishing subjects for study and laboratory practice and in enabling many students to earn part of their educational expenses.

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